Properties of Exponents

Suppose a and b are any NONZERO real numbers, and suppose m and n are any integers.

1. Zero exponent rule: $a^0 = 1$

2. Product rule: $a^m a^n = a^{m+n}$

3. Quotient rule: $\dfrac{a^m}{a^n} = a^{m-n}$

4. Power of a power rule: $(a^m)^n = a^{mn}$

5. Power of a product rule: $(ab)^m = a^m b^m$

6. Power of a quotient rule: $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$

7. Negative exponent rule: $a^{-m} = \dfrac{1}{a^m}$

8. Rule for Negative exponents and fractions

If a and b are nonzero real numbers and m and n are integers, then

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m \quad \text{and} \quad \frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$$

COMMENT: The exponent rules preceding this comment also hold for any *rational* numbers m and n, so long as all the indicated powers are real numbers and no denominator is zero.

Properties of Radicals

9. Converting $a^{n/d}$ to Radical Notation

If a is a real number and d and n are integers for which $\sqrt[d]{a^n}$ is real, then

$$a^{n/d} = \sqrt[d]{a^n}.$$

Moreover $a^{n/d} = \sqrt[d]{a^n} = (\sqrt[d]{a})^n = (a^n)^{1/d} = (a^{1/d})^n$.

COMMENT: Notice the rule $a^{n/d} = \sqrt[d]{a^n}$ is a mnemonic device, i.e. that "*and*" becomes "*dan*."

10. Simplifying "Perfect" nth Roots

n	a	$\sqrt[n]{a}$	$\sqrt[n]{a^n}$
Even	Positive	Positive	a
	Negative	Not a real number	$-a$
Odd	Positive	Positive	a
	Negative	Negative	a

COMMENT: So $\sqrt[n]{a^n} = a$ provided n is not even AND a is not negative.

11. Product rule: $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

12. Quotient rule: $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

13. mth root of an nth root: $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$